**EM 314 – ASSIGNMENT - ODE**

**E/15/202**

**LIYANAGE D.P**

**QUESTION 01**

Here y0 🡪 Initial value of y

h 🡪 step size

end\_val 🡪 y(end value)

start 🡪 y(start)

f 🡪 is the differentiated function

Since the functions are in x or y syms x y used. And values are calculated for x and y

**(A)**

function [] = ForwardEuler(y0,h,end\_val,start)

x0 = start;

n = 0;

syms x y

%y = -0.5\*x^4 + 4\*x^3 - 10\*x^2 + 8.5\*x + 1;

%f = diff(y,x);

%f = x + y;

%f = -2\*x^3 + 12\*x^2 -20\*x + 8.5;

Y\_Euler = y0;

x1 = x0 + n\*h;

while x1 < end\_val

g = subs(f,x,x1);

Y\_Euler = Y\_Euler + h \* subs(g,Y\_Euler);

a = vpa(Y\_Euler,6)

n = n + 1;

x1 = x0 + n\*h;

end

**(B)**

function [] = ImprovedEuler(y0,h,end\_val,start)

x0 = start;

n = 0;

syms x y

%y = -0.5\*x^4 + 4\*x^3 - 10\*x^2 + 8.5\*x + 1;

%f = diff(y,x);

%f = x + y;

%f = -2\*x^3 + 12\*x^2 -20\*x + 8.5;

b = y0;

x1 = x0 + n\*h;

while x1 < end\_val

g = subs(f,x,x1);

hy = b;

b = b + h \* subs(g,b);

x2 = x0 + (n+1)\*h;

k = subs(f,x,x2);

Y\_Avg = (subs(g,hy) + subs(k,b))/2;

Y\_ImproveEuler = hy + h \* Y\_Avg;

b = Y\_ImproveEuler;

a = vpa(Y\_ImproveEuler,6)

n = n + 1;

x1 = x0 + n\*h;

end

**(C)**

function [] = RungeKutta(y0,h,end\_val,start)

x0 = start;

n = 0;

syms x y

%y = -0.5\*x^4 + 4\*x^3 - 10\*x^2 + 8.5\*x + 1;

%f = diff(y,x);

%f = x + y;

%f = -2\*x^3 + 12\*x^2 -20\*x + 8.5;

Y\_RK = y0;

x1 = x0 + n\*h;

while x1 < end\_val

g = subs(f,x,x1);

j = subs(g,Y\_RK);

x2 = (x1 + (x0 + (n+1)\*h))/2;

x3 = x0 + (n+1)\*h;

k0 = h \* j;

g = subs(f,x,x2);

j = subs(g,Y\_RK + 1/2 \* k0);

k1 = h \* j;

g = subs(f,x,x2);

j = subs(g,Y\_RK + 1/2 \* k1);

k2 = h \* j;

g = subs(f,x,x3);

j = subs(g,Y\_RK + k2);

k3 = h \* j;

Y\_Euler = Y\_RK + 1/6 \* (k0 + 2\*k1 + 2\*k2 + k3);

a = vpa(Y\_RK,6)

n = n + 1;

x1 = x0 + n\*h;

end

**QUESTION 02**

Consider and

h 🡪 y

t 🡪 x

then

**(A)**

function [] = ForwardEuler(y0,h,end\_val,start)

x0 = start;

n = 0;

syms x y

f = 10 - 8\*sqrt(y);

Y\_Euler = y0;

x1 = x0 + n\*h;

while x1 < end\_val

g = subs(f,x,x1);

Y\_Euler = Y\_Euler + h \* subs(g,Y\_Euler);

a = vpa(Y\_Euler,6)

n = n + 1;

x1 = x0 + n\*h;

end

**Output:**

ForwardEuler(0,0.2,1.2,0)

|  |  |
| --- | --- |
| y(0) | 0 |
| y(0.2) | 2.0 |
| y(0.4) | 1.73726 |
| y(0.6) | 1.62838 |
| y(0.8) | 1.58665 |
| y(1.0) | 1.57125 |
| y(1.2) | 1.56566 |

ForwardEuler(0,0.1,1.2,0)

|  |  |
| --- | --- |
| y(0) | 0 |
| y(0.1) | 1.0 |
| y(0.2) | 1.2 |
| y(0.3) | 1.32364 |
| y(0.4) | 1.40325 |
| y(0.5) | 1.45558 |
| y(0.6) | 1.4904 |
| y(0.7) | 1.51374 |
| y(0.8) | 1.52947 |
| y(0.9) | 1.5401 |
| y(1.0) | 1.54729 |
| y(1.1) | 1.55217 |
| y(1.2) | 1.55548 |

**(B)**

function [] = ImprovedEuler(y0,h,end\_val,start)

x0 = start;

n = 0;

syms x y

f = 10 - 8\*sqrt(y);

b = y0;

x1 = x0 + n\*h;

while x1 < end\_val

g = subs(f,x,x1);

hy = b;

b = b + h \* subs(g,b);

x2 = x0 + (n+1)\*h;

k = subs(f,x,x2);

Y\_Avg = (subs(g,hy) + subs(k,b))/2;

Y\_ImproveEuler = hy + h \* Y\_Avg;

b = Y\_ImproveEuler;

a = vpa(Y\_ImproveEuler,6)

n = n + 1;

x1 = x0 + n\*h;

end

**Output:**

ImprovedEuler(0,0.2,1.2,0)

|  |  |
| --- | --- |
| y(0) | 0 |
| y(0.2) | 0.868629 |
| y(0.4) | 1.18412 |
| y(0.6) | 1.35257 |
| y(0.8) | 1.44506 |
| y(1.0) | 1.49652 |
| y(1.2) | 1.52535 |

**(C)**

function [] = RungeKutta(y0,h,end\_val,start)

x0 = start;

n = 0;

syms x y

f = 10 - 8\*sqrt(y);

Y\_RK = y0;

x1 = x0 + n\*h;

while x1 < end\_val

g = subs(f,x,x1);

j = subs(g,Y\_RK);

x2 = (x1 + (x0 + (n+1)\*h))/2;

x3 = x0 + (n+1)\*h;

k0 = h \* j;

g = subs(f,x,x2);

j = subs(g,Y\_RK + 1/2 \* k0);

k1 = h \* j;

g = subs(f,x,x2);

j = subs(g,Y\_RK + 1/2 \* k1);

k2 = h \* j;

g = subs(f,x,x3);

j = subs(g,Y\_RK + k2);

k3 = h \* j;

Y\_RK = Y\_RK + 1/6 \* (k0 + 2\*k1 + 2\*k2 + k3);

a = vpa(Y\_RK,6)

n = n + 1;

x1 = x0 + n\*h;

end

**Output:**

RungeKutta(0,0.2,1.2,0)

|  |  |
| --- | --- |
| y(0) | 0 |
| y(0.2) | 0.925929 |
| y(0.4) | 1.24427 |
| y(0.6) | 1.39866 |
| y(0.8) | 1.47706 |
| y(1.0) | 1.51767 |
| y(1.2) | 1.53891 |

**(a)with h = 0.1**

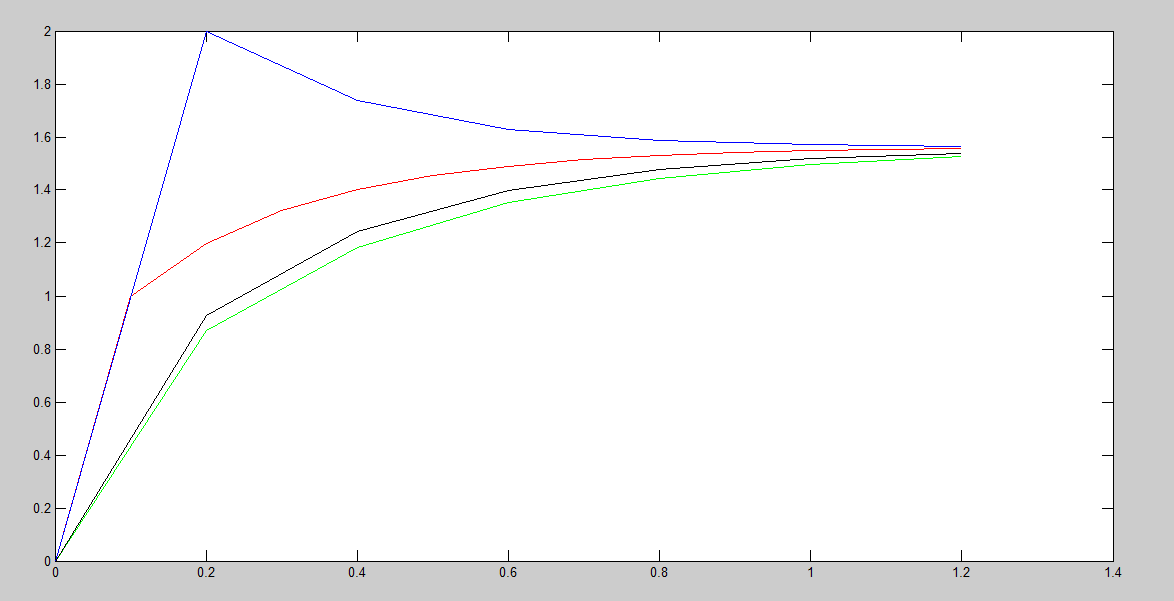
plot([0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2],[0 1.0 1.2 1.32364 1.40325 1.45558 1.4904 1.51374 1.52947 1.5401 1.54729 1.55217 1.55548],'red')

**with h = 0.2**

plot([0 0.2 0.4 0.6 0.8 1.0 1.2],[0 2.0 1.73726 1.62838 1.58665 1.57125 1.56566], 'blue')

**(b)**plot([0 0.2 0.4 0.6 0.8 1.0 1.2],[0 0.868629 1.18412 1.35257 1.44506 1.49652 1.52535],'green')

**(c)**plot([0 0.2 0.4 0.6 0.8 1.0 1.2],[0 0.925929 1.24427 1.39866 1.47706 1.51767 1.53891],'black')

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